

Single and double inclusive forward jet production at the LHC at $\sqrt{s} = 7$ and 13 TeV

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Abstract

We provide a description of the transverse momentum spectrum of single inclusive forward jets produced at the LHC, at the center-of-mass energies of 7 and 13 TeV, using the high energy factorization (HEF) framework. We subsequently study double inclusive forward jet production and, in particular, we calculate contributions to azimuthal angle distributions coming from double parton scattering. We also compare our results for double inclusive jet production to those obtained with the PYTHIA Monte Carlo generator. This comparison confirms that the HEF resummation acts like an initial state parton shower. It also points towards the need to include final state radiation effects in the HEF formalism.

1 Introduction

Processes with jets produced at forward rapidities offer unique access to the corner of phase space where the magnitude of the longitudinal momentum of one of the incoming partons is close to that of the proton, whereas the other parton carries very small fraction of proton's longitudinal momentum, $x \ll 1$. The latter leads to appearance of large logarithms, $\alpha_s \ln(1/x)$, from initial state emissions, which should be resummed, *e.g.* by means of the BFKL equation [1–3], for moderate values of x , or its nonlinear extensions [4–9] if the x is small. The resummation leads to gluon distributions that depend not only on x , but also on the transverse component of gluon's four-momentum, k_t , and the hadronic cross section factorizes into a convolution of such unintegrated gluon distributions and the corresponding off-shell matrix elements. This approach, commonly referred to as k_t -factorization or *high energy factorization* (HEF) [10], will be the basic framework used to study forward jet production in this work. It is worth mentioning that HEF-based approaches to forward jet physics stimulated very interesting recent theoretical developments like the effective TMD approach to dilute-dense collisions [11] or dedicated applications of soft-collinear effective theory (SCET) formalism [12].

Alternatively to the above, one can attempt to calculate predictions for the production of forward jets using general purpose Monte Carlo (MC) programs, such as PYTHIA [13] or HERWIG [14], which are based on the collinear factorization of a $2 \rightarrow 2$ process, supplemented

with an initial- and a final-state parton shower (PS). The advantage of this approach is that it allows one to include a range of potentially important physical effects, such as multi-parton interactions, final-state radiation and non-perturbative corrections. At the same time, however, it lacks formal resummation of terms enhanced with $\alpha_s \ln(1/x)$ and the correct behaviour at low x is only modelled by appropriate initial condition for evolution of the collinear parton density functions.

The collinear-factorization MC tools employing collinear factorization are currently very well developed and have also been successfully used to describe production of forward jets at the LHC (for the latest review see [15]). On the other hand, the recently developed HEF-based tools like AVHLIB [16] and LXJET [17] form a milestone in HEF calculations, they are, however, still at the stage where they can profit from further improvements, as they do not include effects of multi-parton scattering nor a modelling of the final state interactions as for instance is the case in HEF Monte Carlo generator CASCADE [18].

The aim of this work is to make a few steps towards more realistic theoretical description of the forward jet production in the framework of the high energy factorization by investigating the importance of a range of physical effects neglected in earlier analyses. These include: contributions to the hard scattering coming from diagrams with off-shell quarks, contributions from double-parton scattering and the effects of final-state radiation.

The article is organized as follows. We start in Section 2 from studying how the current state-of-the-art HEF framework fares in description of the recent LHC data for the single inclusive forward jet production. Then, in Section 3, we turn to dijet production and study potential importance of several physical components that were not considered in the description of forward jet production so far [19,20]. In particular, in Section 3.1, we use the results from single inclusive jet production to construct double-parton scattering (DPS) contributions to dijet processes and assess their relevance. Then, in Section 3.2, we compare HEF results for forward dijet spectra with those from the PYTHIA MC generator, which include full parton shower, *i.e.* the initial- (ISR) and the final state radiation (FSR). All the above allows us to quantify effects that are currently not included in phenomenological analyses within the HEF framework.

2 Single inclusive forward jet production

The single inclusive jet production is a process which can directly probe partonic content of the proton without a need for large corrections from fragmentation functions. What makes it interesting is the possibility to apply the appropriate formula already at leading order in high energy factorization. This is to be contrasted with collinear factorization, where the $2 \rightarrow 1$ emission vertex vanishes identically and one has to account for higher order corrections either at fixed order of α_s , or with a parton shower.

The single inclusive jet production process can be schematically written as

$$A + B \mapsto a + b \rightarrow \text{jet} + X, \quad (2.1)$$

where A and B are the colliding hadrons, each of which provides a parton, respectively a and b . X corresponds to undetected, real radiation and the beam remnants from the hadrons A and B are understood in the above equation.

The longitudinal kinematic variables read

$$x_1 = \frac{1}{\sqrt{s}} p_{t,\text{jet}} e^{y_{\text{jet}}}, \quad x_2 = \frac{1}{\sqrt{s}} p_{t,\text{jet}} e^{-y_{\text{jet}}}, \quad (2.2)$$

where $s = (p_A + p_B)^2$ is the total squared energy of the colliding hadrons while y_{jet} and p_{jet} are the rapidity and transverse momentum of the leading final state jet, respectively.

The hybrid, high energy factorization formula for the process (2.1), justified for configurations with $x_1 \gg x_2$, at the leading $\ln(1/x)$ accuracy¹, reads [23]

$$\frac{d\sigma}{dy_{\text{jet}} dp_{t,\text{jet}}} = \frac{1}{2} \frac{\pi p_{t,\text{jet}}}{(x_1 x_2 s)^2} \sum_{a,b,c} |\overline{\mathcal{M}_{ab^* \rightarrow c}}|^2 x_1 f_{a/A}(x_1, \mu^2) \mathcal{F}_{b/B}(x_2, p_{t,\text{jet}}^2, \mu^2), \quad (2.3)$$

where \mathcal{F} is a generic notation for the transverse momentum dependent parton density (TMD), which is a function of the longitudinal momentum fraction x , transverse momentum k_t , and the factorization scale μ .

TMDs can be obtained in several ways. In particular, they can be constructed from collinear parton densities via the KMR procedure [24, 25] or, in the approximation in which the gluon dominates over quarks, they can be obtained as solutions of low- x evolution equations [26–31]. The function $x_1 f_{a/A}(x_1, \mu^2)$ is a generic expression for the collinear parton density. The matrix elements $|\overline{\mathcal{M}_{ab^* \rightarrow c}}|^2$ can be obtained via application of the helicity-based formalism [32–34] for off-shell partons or the *parton reggeization approach* [35]. The following channels contribute to the single jet production in HEF approach

$$gg^* \rightarrow g, \quad qq^* \rightarrow q, \quad gq^* \rightarrow q, \quad \bar{q}q^* \rightarrow g. \quad (2.4)$$

The explicit expressions for the corresponding matrix elements are collected in appendix A.

We now turn to predictions for the transverse momentum spectra of the single inclusive forward jets at the LHC. We have performed our calculations at the center-of-mass energies of $\sqrt{s} = 7$ and 13 TeV. The event selection was applied by requiring a leading jet with $p_{t,\text{jet}} > 35$ GeV in the rapidity window of $3.2 < |y_{\text{jet}}| < 4.7$, following the cuts used in the CMS analyses of Refs. [36, 37]. For the on-shell partons, denoted by $x_1 f_{a/A}(x_1, \mu^2)$ in Eq. (2.3), we used the distribution from the CT10 NLO set [38]. For the off-shell partons, $\mathcal{F}_{b/B}(x_2, p_{t,\text{jet}}^2, \mu^2)$, we chose the following set of distributions:

- The “KS nonlinear” unintegrated gluon density [39], which comes from an extension of the BK equation [28, 29] following the prescription of Ref. [40] to include kinematic constraint on the gluons in the chain, non-singular pieces of the splitting functions, as well as contributions from sea quarks. The parameters of the gluon were set by the fit to F_2 data from HERA.
- The “KS linear” gluon [39], determined from linearized version of the equation described above.
- The “KShardscale nonlinear” unintegrated gluon density [41] obtained from the “KS nonlinear” gluon by performing Sudakov resummation of soft emissions between scales given by a hard probe and the scale defined by emission of gluons from the gluonic chain.
- The “KShardscale linear”, determined from linearized version of the equation described above.
- The “DLC2016” (Double Log Coherence) [42] unintegrated parton densities for the gluon and the quarks, determined following the KMR prescription [24, 25]. These distributions are obtained from the standard collinear PDFs supplemented with angular ordering imposed at the last step of evolution and resummation of soft emissions. The Sudakov form factor ensures no emissions between the scale of the gluon transverse momentum, k_t , and the scale of the hard process, μ . The upper cutoff in the Sudakov form factor is chosen such that it imposes angular ordering in the last step of the evolution. The unintegrated parton distributions used in our study are based on CT10 NLO [38].

¹At the NLO level, there exists an extension of this formula for single particle production [21, 22].

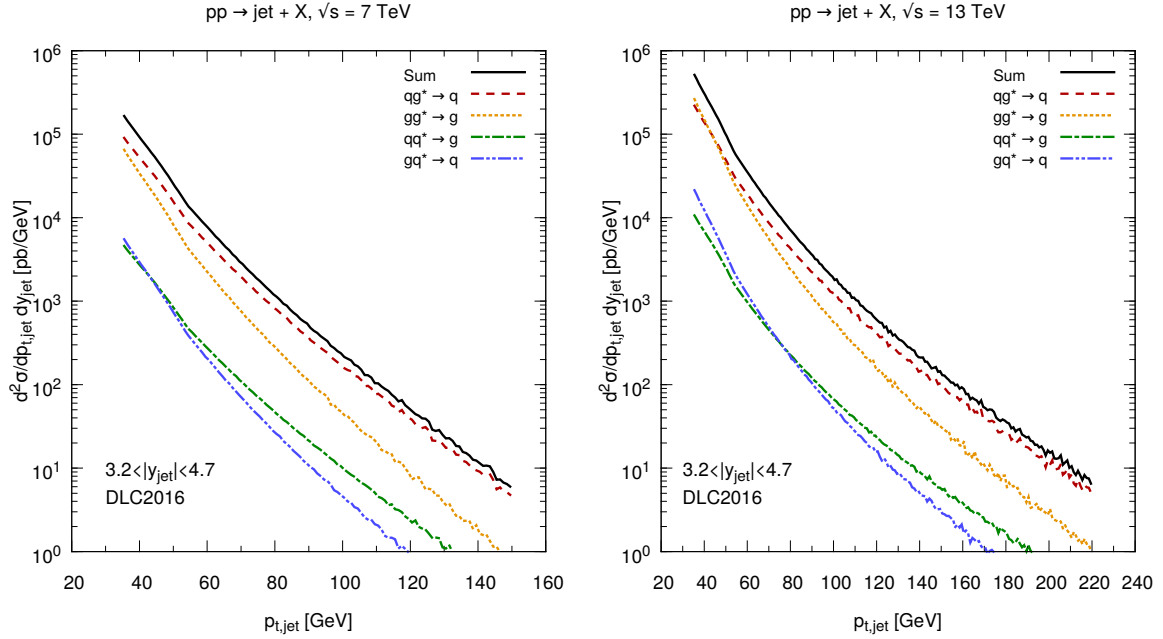


Figure 1: Single inclusive forward jet production. Comparison between different channels contributing to the spectrum of jet's transverse momentum. The results use DLC2016 [42] parametrization for the off-shell partons.

All HEF predictions in this and in the following section were obtained with the FORWARD program [44]. The code implements the hybrid high energy factorization for the single and double jet production and it is capable of using both gluon and quark off-shell parton distributions.

As we see from the above list, all parametrizations, except that of DLC2016, provide only off-shell gluons and neglect off-shell quarks by assuming that their relative contribution is much smaller. The DLC2016 distributions provide the full set of partons and this gives us unique opportunity to verify this assumption.

In Fig. 1 we show predictions for various contributions to the single inclusive jet production obtained using Eq. (2.3) with the DLC2016 off-shell partons. It is evident that the off-shell quark contribution can be indeed effectively neglected and we can proceed just with the off-shell gluons in the initial state. The second interesting observation is that the $qg^* \rightarrow q$ channel gives larger contribution than $gg^* \rightarrow g$ at high transverse momentum while the two channels contribute comparably at lower $p_{t,jet}$.

As the off-shell quark contributions are negligible, it is justified to use all of the off-shell gluon sets listed above as an input for predictions of single inclusive jet spectra. The corresponding results are shown in Fig. 2, where the upper panel shows the absolute distributions, whereas the two lower panels show theory-to-data ratio. We observe good compatibility of the predictions and the 7 and 13-TeV CMS data [36, 37] across a range of unintegrated gluon distributions. We believe that this is a consequence of the TMD factorization applied to low- x physics [45], which states that the same gluon density (if saturation effects are negligible in the considered phase space region) is to be used for the F_2 structure function and for the single inclusive gluon production.

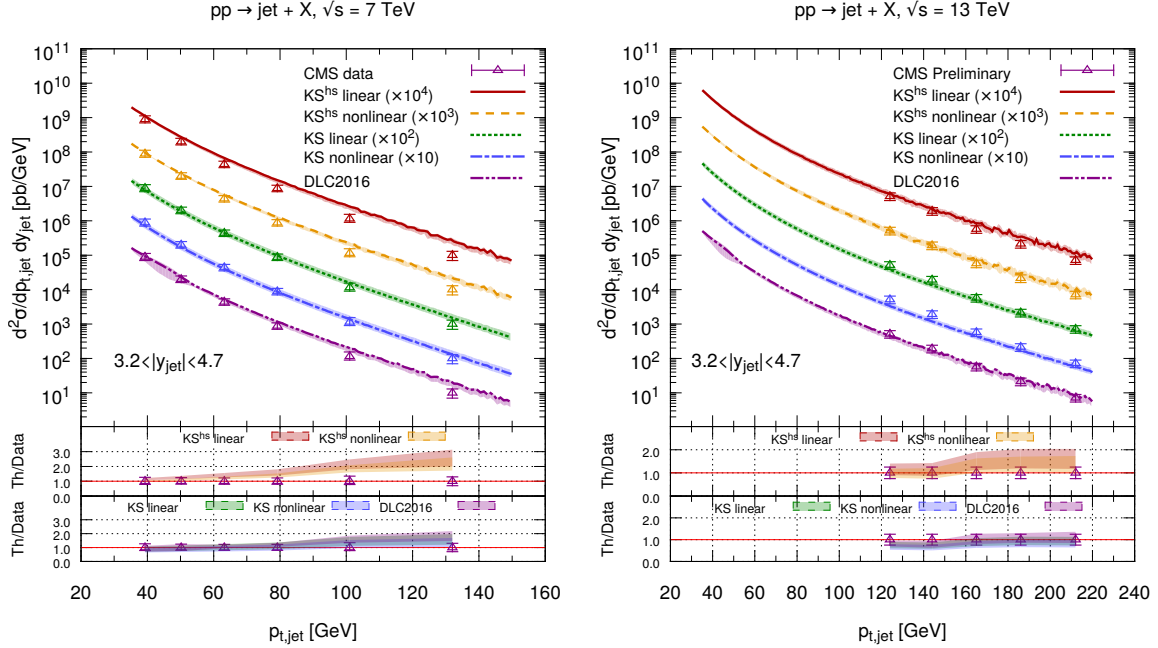


Figure 2: Single inclusive forward jet production. Comparison of predictions for the transverse momentum distributions of a jet with CMS data at 7 [36] and 13 TeV [37]. The bands correspond to different gluon distributions used for calculations. The width of the bands comes from varying the factorization and renormalization scales by factors $\frac{1}{2}$ and 2 around the central value equal to $\mu_F = \mu_R = p_{t,jet}$. In case of KShardscale for 13 TeV, because of limited size of the available grid, we could vary the scale only by the factors $\frac{1}{2}$ and $\frac{3}{2}$. For better visibility, data and predictions with various unintegrated gluons were multiplied by factors 10^n , with $n = 0, \dots, 4$.

3 Forward dijet production

Dijets can be produced either in a *single-parton scattering* (SPS)

$$A + B \mapsto a + b \rightarrow \text{jet} + \text{jet} + X, \quad (3.1)$$

where the partons a and b interact through a $2 \rightarrow 2$ process, or in a *double parton scattering* (DPS)

$$A + B \mapsto a_1 + b_1 + a_2 + b_2 \rightarrow \text{jet} + \text{jet} + X, \quad (3.2)$$

in which each of the incoming hadrons provides two-parton pairs $a_i + b_i$, which in turn undergo two $2 \rightarrow 1$ scatterings. The DPS can be thought of as the single inclusive jet production process of Eq. (2.1) squared.

3.1 Results within HEF formalism

In general, in order to comply with the state of the art of theoretical development, description of the SPS process needs corrections from the improved TMD factorization [11], as it gets contribution from the so-called quadrupole configurations of colour glass condensate (CGC) states and the latter are important in the non-linear domain.

In the present letter, however, we focus on the region of azimuthal distance between the two leading jets, $\Delta\phi$, where the bulk of linear and nonlinear KS densities [39] give comparable results [19] and our aim is just to quantify the potential corrections coming from other physical

effects like DPS contributions and final-state parton shower. Encouraged by the good description of the single inclusive jet production, presented in Section 2, we aim at evaluation of the DPS contribution to inclusive dijet production in order to assess its relative impact with respect to SPS.

The formula for the SPS contribution to forward dijet cross section reads [39, 46]

$$\frac{d\sigma_{\text{SPS}}^{pA \rightarrow \text{dijets}+X}}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \frac{p_{1t} p_{2t}}{8\pi^2 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) |\overline{\mathcal{M}_{ag^* \rightarrow cd}}|^2 \mathcal{F}_{g/A}(x_2, k_t^2) \frac{1}{1 + \delta_{cd}}, \quad (3.3)$$

where

$$x_1 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}), \quad x_2 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}), \quad (3.4)$$

are the fractions of incoming particles' momenta carried by the partons participating in the hard interaction and

$$k_t^2 = |\mathbf{p}_{1t} + \mathbf{p}_{2t}|^2 = p_{1t}^2 + p_{2t}^2 + 2p_{1t}p_{2t} \cos \Delta\phi \quad (3.5)$$

is an imbalance of the transverse momentum of the two leading jets, which, in the HEF formalism is equal to the off-shellness of the incoming gluon. The two leading jets are separated in the transverse plane by the angle $\Delta\phi$.

The expressions for the matrix elements can be found in Refs. [39, 46], while the parton densities at the approximation we are working with are of the same kind as for the single inclusive jet production. Our aim is now to identify and quantify potential corrections to the HEF framework encapsulated in Eq. (3.3).

One of the above may come from double parton scattering [42, 47–52]. In general, the cross section for DPS involves parton density functions which take into account correlations of partons inside the hadrons before the hard scattering [53–56]. However, recent study of Ref. [57] shows that a factorized assumption for DPS is largely valid at high scales ($Q^2 > 10^2 \text{ GeV}^2$). Following this observation, we can therefore write

$$\frac{d\sigma_{\text{DPS}}^{pA \rightarrow \text{dijets}+X}}{dy_1 d^2p_{1t} dy_2 d^2p_{2t}} = \frac{1}{\sigma_{\text{effective}}} \frac{d\sigma}{dy_1 d^2p_{1t}} \frac{d\sigma}{dy_2 d^2p_{2t}}, \quad (3.6)$$

where $\sigma_{\text{effective}} = 15 \text{ mb}$, based on the recent measurement of the LHCb [58, 59] collaboration, which confirmed previous results of D0 [60] and CDF [61]. The explicit expressions for DPS cross sections are given in Appendix B.

The DPS contributions are in general expected to be strong in the low- p_t region of phase space. In order to quantify the role of DPS in forward-forward dijet production, we have calculated the DPS contribution to the azimuthal-angle dependence. Of course, we expect that, in the approximation of Eq. (3.6), where the correlations between incoming partons from different pairs are neglected, the contribution will be just of pedestal type, thus only changing the overall normalization.

In Fig. 3 we show the SPS and the DPS contributions to the azimuthal angle distributions for various cuts on the the hardest jet's transverse momentum, set respectively at 35, 15, 10 and 5 GeV. We see that the relative contribution of DPS increases with lowering the transverse momentum jet cut, but it is significantly smaller than SPS at the experimentally relevant value of 35 GeV. We have checked that the picture looks very similar at 13 TeV.

3.2 HEF vs. collinear factorization

We now turn to the comparison of our HEF predictions for the forward dijet production with the results obtained within collinear factorization². To produce the latter, we have used the

²The preliminary estimate of the result in HEF at 7 TeV has been performed in [43].

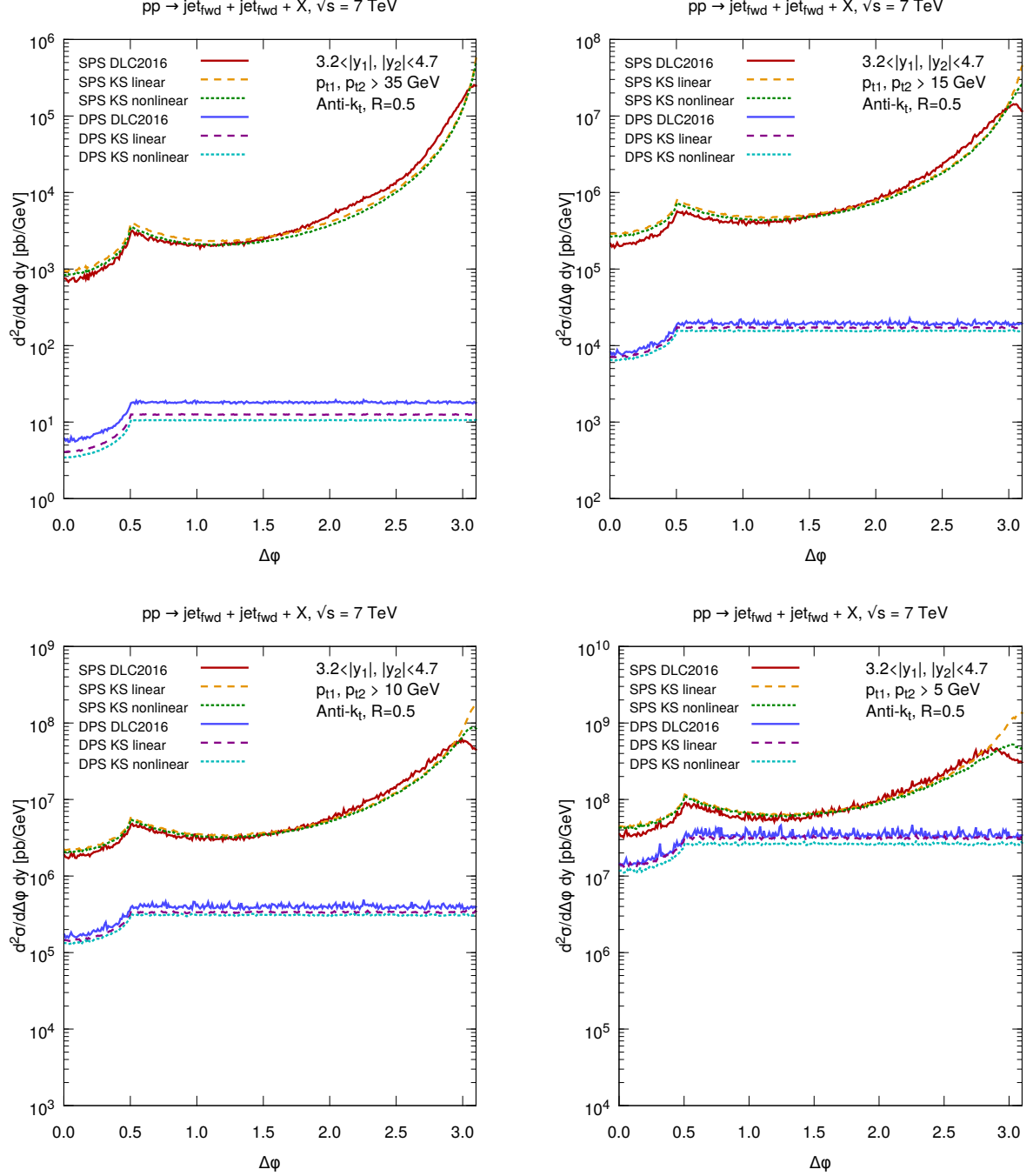


Figure 3: SPS and DPS contribution to forward dijet production for various cuts on transverse momentum of a jet at the LHC at $\sqrt{s} = 7 \text{ TeV}$.

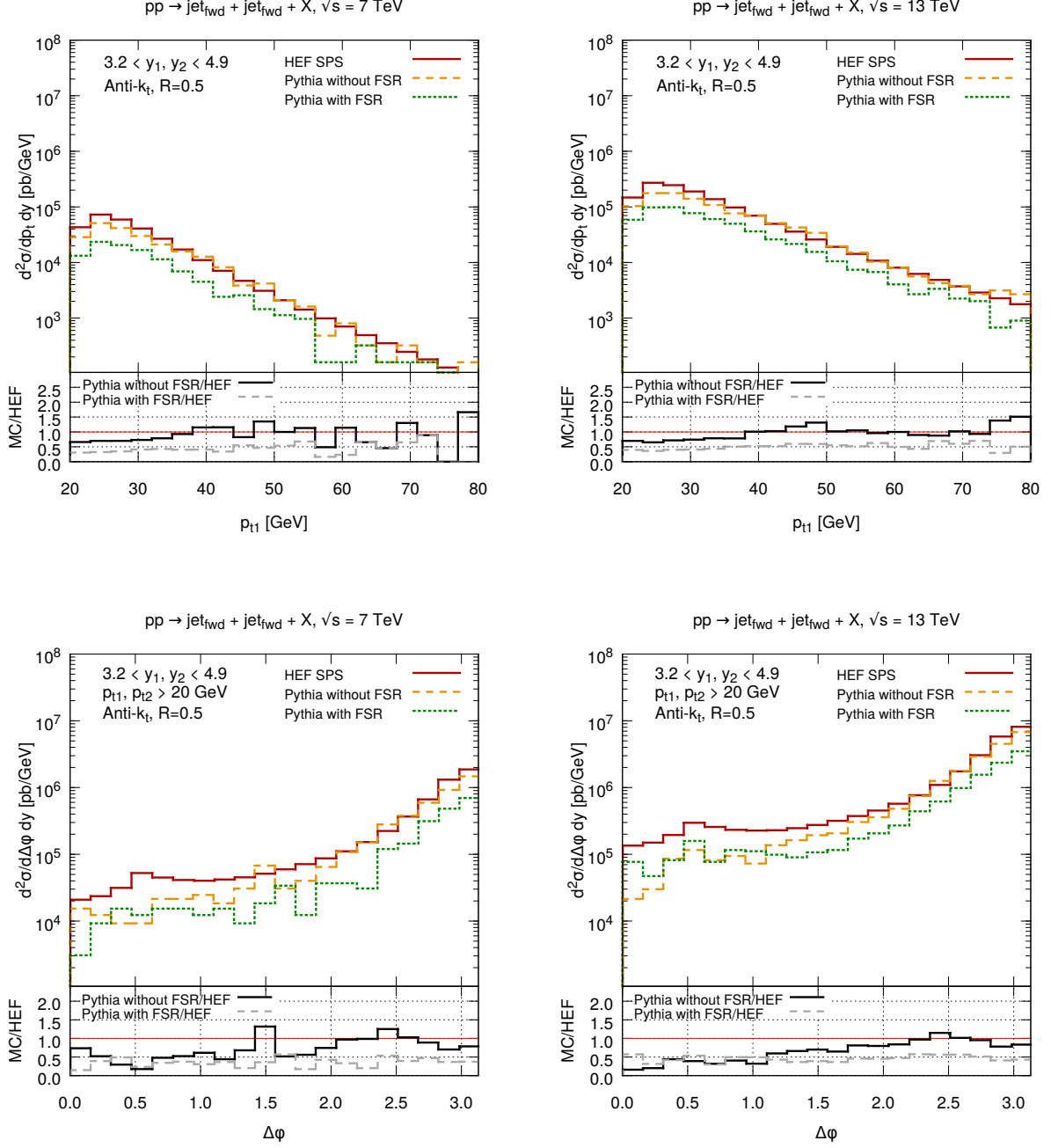


Figure 4: Transverse momentum distribution and azimuthal decorrelation in forward dijet production. Comparison of predictions from high energy factorization, Eq. (3.3) with DLC2016 unintegrated gluon [42], and PYTHIA MC generator. We checked that including MPI has negligible effect on these distributions.

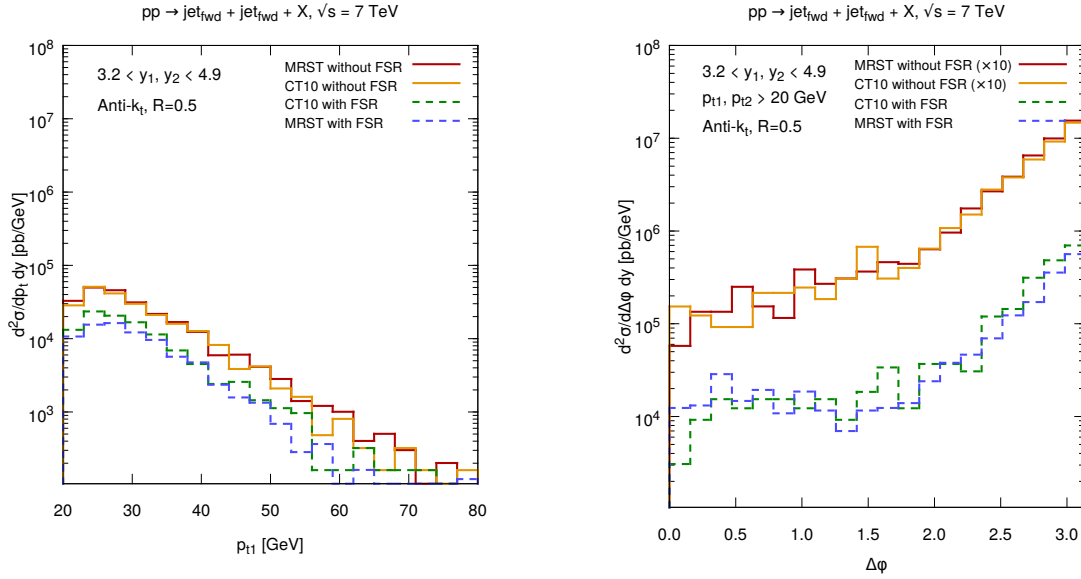


Figure 5: Comparison of results for forward dijet spectra obtained with different PDFs used in PYTHIA MC generator: MRST NLO [62] and CT10 NLO [38].

PYTHIA 8.2 MC generator. To cluster parton level particles into jets, the anti- k_t algorithm [63] was used, as implemented in the FASTJET package [64]. The cuts on the jets' momenta were set to $p_{t,1,2} > 20$ GeV and the rapidity window $3.2 < y_1, y_2 < 4.9$ was imposed to select the jets. The jet radius was set to $R = 0.5$, as in the HEF calculation discussed in preceding subsection.

PYTHIA was set up to generate events including the leading partonic sub-processes. For the comparison with HEF, the CT10 NLO PDFs [38] were used. Runs were performed at two proton-proton collision energies: 7 and 13 TeV. For each energy, two sets of MC data were produced, distinguished by the final state radiation (FSR) option turned on or off.

The comparison between the HEF and the collinear factorization results is shown in Fig. 4. We see that, in the case in which FSR radiation is turned off, the two formalisms agree quite well in description of the p_t spectra in the whole range of considered values. In the case of the azimuthal angle distributions, shown in the lower panel of Fig. 4, the results agree in the region of large and moderate angles and differ in the region of small angles.

We attribute the latter to the effect of different treatment of singularities in the two frameworks. In HEF, matrix element diverges as the two outgoing partons become collinear, see Ref. [11]. This divergence is regularized by the jet algorithm, which is responsible for the kink around $\Delta\phi = 0.5$, seen in the lower panel of Fig. 4. On the other hand, in the case of PYTHIA, the shape of the distribution is a result of initial state radiation generated via parton shower matched with the collinear matrix element. Since the collinear matrix elements have different singularity structure than the HEF matrix elements this leads to different results at small $\Delta\phi$ [11, 46].

In Fig. 5 we show a comparison between distributions obtained with PYTHIA but using two different PDF sets. As we see, both sets give similar results, hence, the qualitative differences between PYTHIA and HEF, seen in Fig. 4, cannot be attributed to a choice of PDFs.

In Fig. 4, we also observe that turning on the FSR in PYTHIA leads to change in normalization of both the p_t and $\Delta\phi$ distributions. The spectra decrease by factor ~ 2 for moderate and large p_t , as well as $\Delta\phi$, values. Low- p_t and low- $\Delta\phi$ parts of the distributions are almost not affected by FSR.

The observed difference in normalization of the transverse momentum spectra can be explained by the energy loss of the leading hard parton that happens readily via FSR parton shower emissions. For a significant fraction of events, this leads to the situation in which the parton originating from the hard collision splits into two partons separated by an angle sufficient to produce two lower- p_t jets. This mechanism takes the high- p_t events from the tail of the spectrum without FSR and moves them to the region below the jet cut. Hence, they effectively do not contribute to the observables shown in Fig. 4.

Finally, we mention that we have checked explicitly that the picture of Fig. 4 persists if PYTHIA events are supplemented with multi-parton interactions (MPI). Hence, forward dijet production in the collinear factorization framework is weakly sensitive to MPIs, which is consistent with the negligible effect of DPS in HEF, which we demonstrated in Section 2.

4 Conclusions

In this letter, we have studied double and single inclusive forward jet production at the LHC within two formalisms: the high energy factorisation (HEF) and the collinear factorization.

We have demonstrated that the HEF framework describes well the single inclusive jet production at the LHC, at the center-of-mass energies of 7 and 13 TeV, and the main uncertainty comes from the unintegrated parton distributions. In this context, we have observed that the contribution from the off-shell quarks is negligible for forward jet production.

We have also explicitly shown that, for typical experimental cuts used in inclusive dijet production processes, the double parton scattering effects can be safely neglected. Finally, our study shows that the effect of the final state radiation is not negligible and it leads to change of normalization of differential distributions in forward dijet production.

Acknowledgments

We are grateful to Andreas van Hameren for careful reading of the manuscript and numerous useful suggestions. We also appreciate comments from Piotr Kotko and Hannes Jung. The work of M.B. and K.K. has been supported by Narodowe Centrum Nauki (NCN) with Sonata Bis grant DEC-2013/10/E/ST2/00656. M.D. acknowledges hospitality of Instytut Fizyki Jadrowej, where the project was initiated, as well as the support from the NCN grant DEC-2013/10/E/ST2/00656.

A Matrix elements

The matrix elements squared for three-parton processes, used in the calculation of the single inclusive forward jet distributions in Section 2 read

- $gg^* \rightarrow g$

$$\overline{|\mathcal{M}_{gg^* \rightarrow g}|^2} = 4g_s^2 \frac{C_A}{N_c^2 - 1} \frac{(k \cdot q)^2}{k_t^2}, \quad (\text{A.1})$$

- $qq^* \rightarrow q$

$$\overline{|\mathcal{M}_{qq^* \rightarrow q}|^2} = 4g_s^2 \frac{C_F}{N_c^2 - 1} \frac{(k \cdot q)^2}{k_t^2}, \quad (\text{A.2})$$

- $gq^* \rightarrow q$

$$\overline{|\mathcal{M}_{gq^* \rightarrow q}|^2} = g_s^2 \frac{C_F}{N_c^2 - 1} (k \cdot q), \quad (\text{A.3})$$

- $\bar{q}q^* \rightarrow g$

$$|\overline{\mathcal{M}_{\bar{q}q^* \rightarrow g}}|^2 = g_s^2 \frac{C_F}{N_c} (k \cdot q). \quad (\text{A.4})$$

In the above, k and q are the momenta of the off-shell and on-shell partons, respectively. k_t is the transverse component of the off-shell momentum, g_s is a strong coupling and C_i is a colour factor of the emitter: C_F for a quark and C_A for a gluon.

B Double-parton scattering formulae

The explicit expression for the DPS contribution in the factorized approximation reads

$$\begin{aligned} \frac{d\sigma_{\text{DPS}}}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} &= \frac{1}{\sigma_{eff}} \frac{\pi}{8} \frac{p_{1t}}{(x_1 x_2 s)^2} \frac{p_{2t}}{(\bar{x}_1 \bar{x}_2 s)^2} \\ &\times \left(|\overline{\mathcal{M}_{gg^* \rightarrow g}}|^2 x_1 f_{g/A}(x_1) + \sum_{i=1}^{n_f} |\overline{\mathcal{M}_{qg^* \rightarrow q}}|^2 x_1 f_{q(i)/A}(x_1) \right) \\ &\times \left(|\overline{\mathcal{M}_{gg^* \rightarrow g}}|^2 \bar{x}_1 \bar{f}_{g/A}(\bar{x}_1) + \sum_{i=1}^{n_f} |\overline{\mathcal{M}_{qg^* \rightarrow q}}|^2 \bar{x}_1 \bar{f}_{q(i)/A}(\bar{x}_1) \right) \\ &\times \mathcal{F}_{g^*/B}(x_2, p_{1t}^2) \bar{\mathcal{F}}_{g^*/B}(\bar{x}_2, p_{2t}^2) \theta(1 - x_1 - \bar{x}_1) \theta(1 - x_2 - \bar{x}_2), \end{aligned} \quad (\text{B.1})$$

where, in order to be compatible with the SPS formula (3.3) we introduced an auxiliary azimuthal angle between the final state jets, $\Delta\phi$. The notation follows that of Eq. (3.3) except that now, each of the incoming particles provides a pair of partons, whose energy fractions are given by x_1 and \bar{x}_1 for hadron A , and x_2 and \bar{x}_2 for hadron B . The theta functions guarantee that a pair of partons from a single hadron does not carry more than 100% of the hadron's energy.

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